

2-6 Study Guide and Intervention

Algebraic Proof

Algebraic Proof A list of algebraic steps to solve problems where each step is justified is called an **algebraic proof**. The table shows properties you have studied in algebra.

The following properties are true for any real numbers a , b , and c .

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then, $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$ and $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

Example Solve $6x + 2(x - 1) = 30$. Write a justification for each step.

Algebraic Steps

$$\begin{aligned}
 6x + 2(x - 1) &= 30 \\
 6x + 2x - 2 &= 30 \\
 8x - 2 &= 30 \\
 8x - 2 + 2 &= 30 + 2 \\
 8x &= 32 \\
 \frac{8x}{8} &= \frac{32}{8} \\
 x &= 4
 \end{aligned}$$

Properties

Original equation or Given
 Distributive Property
 Substitution Property of Equality
 Addition Property of Equality
 Substitution Property of Equality
 Division Property of Equality
 Substitution Property of Equality

Exercises

Complete each proof.

1. Given: $\frac{4x + 6}{2} = 9$

Prove: $x = 3$

Proof:

Statements	Reasons
a. $\frac{4x + 6}{2} = 9$	a. GIVEN
b. $2\left(\frac{4x + 6}{2}\right) = 2(9)$	b. Mult. Prop.
c. $4x + 6 = 18$	c. SIMPLIFY
d. $4x + 6 - 6 = 18 - 6$	d. - Prop of =
e. $4x = 12$	e. SIMPLIFY
f. $\frac{4x}{4} = \frac{12}{4}$	f. Div. Prop.
g. $x = 3$	g. SIMPLIFY

2. Given: $4x + 8 = x + 2$

Prove: $x = -2$

Proof:

Statements	Reasons
a. $4x + 8 = x + 2$	a. GIVEN
b. $4x + 8 - x = x + 2 - x$	b. - Prop of =
c. $3x + 8 = 2$	c. SIMPLIFY
d. $(-3)3x + 8 = 2(-3)$	d. Subtr. Prop.
e. $3x = -6$	e. SIMPLIFY
f. $\frac{3x}{3} = \frac{-6}{3}$	f. \div Prop of =
g. $x = -2$	g. SIMPLIFY

Solve each equation. Write a reason for every step.

1. $4x = 12x + 32$

① $4x = 12x + 32$	① GIVEN
② $4x - (32) = 12x + 32 - 32$	② -PROP OF =
③ $4x - 32 = 12x$	③ SIMPLIFY
④ $4x - 32 - (4x) = 12x - 4x$	④ -PROP OF =
⑤ $-32 = 8x$	⑤ SIMPLIFY
⑥ $\frac{-32}{8} = \frac{8x}{8}$	⑥ \div PROP OF =
⑦ $-4 = x$	⑦ SIMPLIFY

2. $28 + 12x = 8x - 4$

① $28 + 12x = 8x - 4$	① GIVEN
② $28 + 12x - (28) = 8x - 4 - (28)$	② -PROP OF =
③ $12x = 8x - 32$	③ SIMPLIFY
④ $12x - (8x) = 8x - 32 - 8x$	④ -PROP OF =
⑤ $4x = -32$	⑤ SIMPLIFY
⑥ $\frac{4x}{4} = \frac{-32}{4}$	⑥ \div PROP OF =
⑦ $x = -8$	⑦ SIMPLIFY

3. $60x + 153 = 9x + 51$

① $60x + 153 = 9x + 51$	① GIVEN
② $60x + 153 - (153) = 9x + 51 - (153)$	② -PROP OF =
③ $60x = 9x - 102$	③ SIMPLIFY
④ $60x - 9x = 9x - 102 - 9x$	④ -PROP OF =
⑤ $51x = -102$	⑤ SIMPLIFY
⑥ $\frac{51x}{51} = \frac{-102}{51}$	⑥ \div PROP OF =
⑦ $x = -2$	⑦ SIMPLIFY

4. $-4x + 10 = -5x + 18$

① $-4x + 10 = -5x + 18$	① GIVEN
② $-4x + 10 + 4x = -5x + 18 + 4x$	② + PROP OF =
③ $10 = -x + 18$	③ SIMPLIFY
④ $10 - 18 = -x + 18 - 18$	④ -PROP OF =
⑤ $-8 = -x$	⑤ SIMPLIFY
⑥ $(-8)(-1) = (-x)(-1)$	⑥ \times PROP OF =
⑦ $8 = x$	⑦ SIMPLIFY
⑧ $x = 8$	⑧ SYMMETRIC PROP

5. $-3(x + 2) = 16 - x$

① $-3(x + 2) = 16 - x$	① GIVEN
② $-3x - 6 = 16 - x$	② DISTRIBUTIVE
③ $-3x - 6 + x = 16 - x + x$	③ + PROP OF =
④ $-2x - 6 = 16$	④ SIMPLIFY
⑤ $-2x - 6 + 6 = 16 + 6$	⑤ + PROP OF =
⑥ $-2x = 22$	⑥ SIMPLIFY
⑦ $\frac{-2x}{-2} = \frac{22}{-2}$	⑦ \div PROP OF =
⑧ $x = -11$	⑧ SIMPLIFY

6. $-x - 2(9 - 8x) = 12$

① $-x - 2(9 - 8x) = 12$	① GIVEN
② $-x - 18 + 16x = 12$	② DISTRIBUTIVE
③ $-18 + 15x = 12$	③ COMBINE LIKE TERMS/SIMPLIFY
④ $-18 + 15x + 18 = 12 + 18$	④ + PROP OF =
⑤ $15x = 30$	⑤ SIMPLIFY
⑥ $\frac{15x}{15} = \frac{30}{15}$	⑥ \div PROP OF =
⑦ $x = 2$	⑦ SIMPLIFY

8. $\frac{1}{4}x + 10 = 2$

① $\frac{1}{4}x + 10 = 2$	① GIVEN
② $\frac{1}{4}x + 10 - 10 = 2 - 10$	② -PROP OF =
③ $\frac{x}{4} = -8$	③ SIMPLIFY
④ $4 \cdot \frac{x}{4} = -8 \cdot 4$	④ \times PROP OF =
⑤ $x = -32$	⑤ SIMPLIFY

9. $6(x - 6) = x(16 - 7)$

① $6(x - 6) = x(16 - 7)$	① GIVEN
② $6x - 36 = 16x - 7x$	② DISTRIBUTIVE
③ $6x - 36 = 9x$	③ SIMPLIFY
④ $6x - 36 - 6x = 9x - 6x$	④ -PROP OF =
⑤ $-36 = 3x$	⑤ SIMPLIFY
⑥ $\frac{-36}{3} = \frac{3x}{3}$	⑥ \div PROP OF =
⑦ $-12 = x$	⑦ SIMPLIFY
⑧ $x = -12$	⑧ SYMMETRIC PROP